11th IIT PAPER solutions _ 15.07.2019

Physics

1 (a)

Let $F \propto P^x V^y T^z$

By substituting the following dimensions:

$[P] = [ML^{-1}T^{-2}][V] = [LT^{-1}], [T] = [T]$

and comparing the dimension of both sides

$x = 1, y = 2, z = 2$, so $F = PV^2T^2$

2 (a)

Volume of cube $= a^3$

Surface area of cube $= 6a^2$

according to problem $a^3 = 6a^2 \Rightarrow a = 6$

$\therefore V = a^3 = 216\, units$

4 (b)

5 (b)

6 (c)

CR is known as time constant $CR = [T]$

7 (a)

Quantities having different dimensions can only be divided or multiplied but they cannot be added or subtracted

8 (a)

$Bxt$is unitless. $\therefore$ Unit of B is$m^{-1}s^{-1}$

10 (b)

$$mv = kg\left(\frac{m}{\text{sec}}\right)$$

11 (c)

12 (b)

Time constant in an $R - C$ circuit

$\tau = R - C$

$[\tau] = [R][C]$

$= [\text{ML}^2\text{T}^{-3}\text{A}^{-2}][\text{M}^{-1}\text{L}^{-2}\text{T}^4\text{A}^2]$

$= [\text{M}^0\text{L}^0\text{T}]$

13 (d)

R=8.3 J/K-mol

$n_1u_1 = n_2u_2$

$$\therefore\ n_2 = \frac{n_1u_1}{u_2}$$

$$= \frac{8.3\text{J/K} - \text{mol}}{\text{atm L/K} - \text{mol}}$$

$$= \frac{8.3\text{J/K} - \text{mol}}{(1.013 \times 10^5\text{N/m}^2)(10^{-3}\text{m}^3)/\text{K} - \text{mol}}$$

$$= \frac{8.12}{10^2} = 0.0812$$

$\therefore$ 8.3 J/K-mol=0.0812 atm L/K-mol

14 (a)

From Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$\Rightarrow \left[\frac{1}{4\pi\varepsilon_0}\right] = \frac{[F \times r^2]}{[q]^2}$$

$$= \frac{[\text{newton}][\text{metre}]^2}{[\text{coulomb}]^2}$$

$$= \text{Nm}^2\text{C}^{-2}$$

15 (a)

Required percentage error

$$= 2 \times \frac{0.01}{15.12} \times 0 + \frac{0.001}{10.15} \times 10 = 4 + 1 = 5$$

Chemistry solutions

1 (b)

Ar and Ca^{2+} are isoelectronic species as they have same number of electrons, *i.e.*, 18.

2 (a)

Tritium is the isotope of hydrogen. Its composition is as follows :

1 electron, 1 proton and 2 neutrons

3 (b)

$E = \frac{hc}{\lambda}$, h and c for both causes are same so,

$$\frac{E_1}{E_2} = \frac{\lambda_2}{\lambda_1} = \frac{16000}{8000}$$

$$E_1 = 2E_2$$

4 (c)

When $n = 4$ and $x = 5$ then electronic configuration can be written as

$(4-1)s^2(4-1)p^6(4-1)d^5 4s^2$

This electronic configuration represents Mn and its atomic number is 25. Hence, number of protons are 25 in its nucleus.

5 (c)

Isotones are species which have equal number of neutrons.

Neutrons in $_{19}K^{39} = 39 - 19 = 20$

Neutrons in $_{20}Ca^{40} = 40 - 20 = 20$

6 (d)

The mass number =atomic number + number of neutron

Atomic number=no. of proton

=no. of electron (for an atom)

So, mass number =18+20=38

7 (c)

Bohr's theory is applicable to unielectron atom or ion only.

8 (a)

Wave number of spectral line in emission spectrum of hydrogen,

$$\bar{v} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \text{... (i)}$$

Given, $\bar{v} = \frac{8}{9}R_H$

On putting the value of $\bar{v}$ in Eq. (i), we get

$$\frac{8}{9} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{8}{9} = \frac{1}{(1)^2} - \frac{1}{n_2^2}$$

$$\frac{8}{9} - 1 = -\frac{1}{n_2^2}$$

$$\frac{1}{3} = \frac{1}{n_2}$$

$$\therefore \quad n_2 = 3$$

Hence, electron jumps from $n_2 = 3$ to $n_1 = 1$

9 (d)

Angular momentum of an electron

$= mvr = \frac{nh}{2\pi}$ (n is orbit number)

in 5th orbit $= \frac{5h}{2\pi} = \frac{2.5h}{\pi}$

10 (d)

Hydrogen spectrum is an emission spectrum. It shows the presence of quantized energy levels in hydrogen atom.

11 (d)

Bohr radius for nth orbit $= 0.53 \times \frac{n^2}{Z}$

Where, Z =atomic number

$\therefore$ Bohr radius of 2nd orbit of $Be^{3+} = \frac{0.53 \times (2)^3}{4}$

$= 0.53$ Å

(d) Bohr radius of 1st orbit of H$= \frac{0.53 \times (1)^2}{1}$

Hence, Bohr's radius of 2nd orbit of Be^{3+} is equal to that of first orbit of hydrogen.

12 (a)

λ for visible light is in the range of 400 to 780 nm.

$E = \frac{hc}{\lambda}$.

This, it is in the range of electron volt (eV).

13 (a)

The value of Rydberg constant is 10,9678 cm^{-1}.

14 (d)

According to Bohr model,

Radius of hydrogen atom

$(r_n) = \frac{0.529 \times n^2}{Z}$ Å

Where, n= number of orbit

Z= atomic number

$$r_3 = \frac{0.529 \times (3)^2}{1} = 4.761\text{Å}$$

15 (c)

Kinetic energy $= \frac{1}{2} mv^2$,

Potential energy $= \frac{-e^2}{r}$

But, $mv^2 = \frac{e^2}{r}$

KE$= \frac{1}{2}\frac{e^2}{r}$

Total energy =KE+PE

$$= \frac{1}{2}\frac{e^2}{r} - \frac{e^2}{r} = \frac{e^2}{r}\left(\frac{1}{2} - 1\right) = \frac{-e^2}{2r}$$

Mathematics paper solution

Exam Date: 15/07/2019 (I.I.T) By: M.K. sir

Hints and solution 11th Science Logarithm

Q.31 $\log_{2\sqrt{2}} 32\sqrt[5]{4} = \log_{(2^{3/2})}\left(2^5 \cdot 4^{\frac{1}{5}}\right) = \log_{2^{3/2}}\left(2^{(5+\frac{2}{5})}\right) = \frac{2}{3}\cdot\frac{27}{5}\log_2 2$

$= \frac{2}{3}\cdot\frac{27}{5} = \frac{18}{5} = 3.6$ Ans option (a)

Q.32. given $x = \log_3 5$

$\Rightarrow \frac{1}{x} = \log_5 3$

$\Rightarrow \frac{1}{x} = \frac{1}{2}\cdot\log_5 9$

$y = \log_{17} 25 = \log_{17} 5^2$

$\therefore \frac{1}{y} = \frac{1}{2}\log_5 17$

$\therefore \frac{1}{y} > \frac{1}{x} \Rightarrow x > y$ Ans option (b)

Q.33 $y = 2^{\frac{1}{\log_x 4}}$

$\Rightarrow \log_2 y = \frac{1}{\log_x 4} = \log_4 x = \frac{1}{2}\log_2 x$

$\therefore 2\log_2 y = \log_2 x$

$\Rightarrow \therefore y^2 = x$ Ans option (a)

Q.34. $\log_2(3x-2) = \log_{1/2} x$

$\Rightarrow \log_2(3x-2) = -\log_2 x = \log_2 x^{-1}$

$\Rightarrow (3x-2) = x^{-1}$

$\Rightarrow 3x^2 - 2x = 1 \Rightarrow x = 1$ ✓ or $\frac{-1}{3}$ ✗

But given question are meaningful if $x > 2/3$.

Hence $x = 1$ Ans option (d)

Q.35 $\log_3 (x-2) \le 2$ $\quad [\because (x-2) > 0]$

$\Rightarrow 0 < (x-2) \le 3^2$

$\Rightarrow 2 < x \le 9+2$

$\Rightarrow \boxed{2 < x \le 11}$ $\quad$ option (b)

Q.36 $\log_{0.2} |x-3| \ge 0$

$\Rightarrow 0 < |x-3| \le (0.2)^0$

$\Rightarrow -1 \le x-3 \le 1 \;\&\; x-3 \ne 0$

$\Rightarrow \boxed{2 \le x \le 4} \;\&\; x \ne 3$ $\quad$ option (b)

$\therefore x \in [2,4] - \{3\}$

Q.37 $6(\log_x 2 - \log_4 x) + 7 = 0$

$\Rightarrow 6\left(\log_x 2 - \frac{1}{2}\log_2 x\right) + 7 = 0$ $\quad$ put, $\log_2 x = y$

$\Rightarrow 6\left(\frac{1}{y} - \frac{1}{2}\cdot y\right) + 7 = 0$ $\quad \therefore \log_x 2 = \frac{1}{y}$

$\Rightarrow 6\left(\frac{2-y^2}{2y}\right) + 7 = 0$

$\Rightarrow 6 - 3y^2 + 7y = 0$

$\Rightarrow 3y^2 + 2y - 9y - 6 = 0$

$\Rightarrow (y-3)(3y+2) = 0$

$\Rightarrow y = 3$ or $-2/3$

$\therefore \log_2 x = 3$ or $\log_2 x = -2/3$

$\therefore x = 2^3$ or $\boxed{x = 2^{-2/3}}$

$\boxed{x = 8}$

$\therefore$ option (c).

Q.38. $\log_2 (x-1) > 4$

$\Rightarrow (x-1) > 2^4$

$\Rightarrow x > 17$ Ans option (c)

Q.39 $\log_4 (2\times 4^{x-2} - 1) + 4 = 2x$

$\Rightarrow \log_4 (2\times 4^{x-2} - 1) = 2x - 4$

$\Rightarrow 2\times 4^{x-2} - 1 = 4^{2x-4} = 4^{2(x-2)} = (4^{(x-2)})^2$

put $4^{x-2} = y$

$\Rightarrow 2y - 1 = y^2$

$\Rightarrow y^2 - 2y + 1 = 0$

$\Rightarrow (y-1)^2 = 0 \Rightarrow y = 1$

$\therefore 4^{x-2} = 1 = 4^0$

$\Rightarrow x - 2 = 0$

$\Rightarrow x = 2$ Ans option (c)

Q.40 $\log_{0.5} \frac{3-x}{x+2} < 0$

$\Rightarrow \frac{3-x}{x+2} > (0.5)^0$

$\Rightarrow \frac{3-x}{x+2} > 1$

$\Rightarrow \frac{3-x-x-2}{x+2} > 0$

$\Rightarrow \frac{-2x+1}{x+2} > 0$

$\Rightarrow \frac{2x-1}{x+2} < 0$

$\therefore 2x-1 < 0$ and $x+2 > 0$

$\Rightarrow x < \frac{1}{2}$ and $x > -2$

$\therefore -2 < x < \frac{1}{2}$ Ans

option (b)

Single Correct Answer Type

41 (2)

Since, $A + B + C = \pi$

$\Rightarrow a = \pi - (B + C)$

We have, $\cos A = \cos B \cos C$

$\Rightarrow \cos[\pi - (B + C)] = \cos B \cos C$

$\Rightarrow -\cos(B + C) = \cos B \cos C$

$\Rightarrow -[\cos B \cos C - \sin B \sin C] = \cos B \cos C$

$\Rightarrow \sin B \sin C = 2 \cos B \cos C$

$\Rightarrow \tan B \tan C = 2$

42 (1)

$x^2 + y^2 + z^2 = r^2 \sin^2 \theta \cos^2 \phi + r^2 \sin^2 \theta \sin^2 \phi + r^2 \cos^2 \theta$

$= r^2 \sin^2 \theta(\cos^2 \phi + \sin^2 \phi) + r^2 \cos^2 \theta$

$= r^2 \sin^2 \theta + r^2 \cos^2 \theta$

$= r^2$

43 (3)

We have, $x \cos \theta = y \cos\left(\theta + \frac{2\pi}{3}\right)$

$= z \cos\left(\theta + \frac{4\pi}{3}\right) = k$ (say)

$\Rightarrow \cos \theta = \frac{k}{x}, \cos\left(\theta + \frac{2\pi}{3}\right) = \frac{k}{y}$

and $\cos\left(\theta + \frac{4\pi}{3}\right) = \frac{k}{z}$

$\therefore \frac{k}{x} + \frac{k}{y} + \frac{k}{z} = \cos \theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)$

$= \cos \theta - \cos\left(\frac{\pi}{3} - \theta\right) - \cos\left(\frac{\pi}{3} + \theta\right)$

$= \cos \theta - 2 \cos \frac{\pi}{3} \cos \theta = 0$

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

44 (3)

Given that, $\sin\theta + \cos\theta = m$ ... (i)

and $\sec\theta + \operatorname{cosec}\theta = n$ ... (ii)

Now, $n(m+1)(m-1) = n(m^2-1)$

$= (\sec\theta + \operatorname{cosec}\theta) 2\sin\theta\cos\theta$ $(\because m^2 = 1 + 2\sin\theta\cos\theta)$

$= \dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} 2\sin\theta\cos\theta$

$= 2m$ [from Eq.(i)]

45 (1)

We have, $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$$= \frac{\dfrac{1}{\sqrt{x(x^2+x+1)}} + \dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1 - \dfrac{1}{\sqrt{x(x^2+x+1)}} . \dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}}$$

$$= \frac{(1+x)\sqrt{x^2+x+1}}{\sqrt{x}.x(x+1)}$$

$= \sqrt{x^{-3} + x^{-2} + x^{-1}} = \tan\gamma$ (given)

$\therefore\ \alpha + \beta = \gamma$