

11TH IIT MATHEMATICS PAPER HINTS AND SOLUTIONS 30.09.2019

31. $(0.1)Q = \frac{1}{2}(A - A^T)$

32. $BC = I$, $\text{tr}(A) = 2 + 1 = 3$

$$\text{Tr}(A) + \text{tr}\left(\frac{ABC}{2}\right) + \text{tr}\left(\frac{A(BC)^2}{4}\right) + \dots$$

$$= \text{tr} A + \text{tr}\left(\frac{A}{2}\right) + \text{tr}\left(\frac{A}{4}\right) + \dots$$

$$= 3 + \frac{3}{2} + \frac{3}{4} + \dots$$

$$= 3\left(1 + \frac{1}{2} + \frac{1}{4} + \dots\right) = 3\left(\frac{1}{1 - \frac{1}{2}}\right) = 6$$

33. $(3A + 4B^T)^T = 3A^T + 4B = \begin{bmatrix} 7 & 0 \\ -10 & 6 \\ 17 & 31 \end{bmatrix}$ ---(1)

$$2B - 3A^T = \begin{bmatrix} -1 & 18 \\ 4 & -6 \\ -5 & -7 \end{bmatrix}$$
 ---(2)

$$1+2 \Rightarrow 6B = \begin{bmatrix} 6 & 18 \\ -6 & 0 \\ 12 & 24 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 2 & 4 \end{bmatrix}$$

34. $|A \text{ dj}(A \text{ dj} A)| = |A|^{(n-1)^2}$
 $|A \text{ dj}(A \text{ dj}(A^2))| = |A^2|^4 = |A|^8$

35. $PP^T = I$, $Q = PAP^T$
 $P^T Q^{2005} P = P^T(PAP^T)(PAP^T)\dots(2005 \text{ times } P)$
 $= (P^T P)(A)(P^T P)(A)\dots(A) \text{ 2005 times}$
 $= I(A)I(A)\dots\text{2005 times}$
 $= A^{2005}$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \Rightarrow A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

36. $f(x) = x^2 + 4x - 5 \Rightarrow$

$$f(A) = A^2 + 4A - 5I$$

$$f(A) = \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$$

$$37. B = \text{Adj } A, \quad C = 5A \quad |A| = 1(3) + 1(6) + 1(-4) = 5$$

$$\frac{|Adj B|}{|C|} = \frac{|Adj(Adj A)|}{|5A|} = \frac{|A|^3}{5^3} = \frac{5^3}{5^3} = 1$$

$$38. \begin{vmatrix} x^2 + x & 31 - 1 & -x + 3 \\ 2x + 1 & 2 + x^2 & x^3 - 3 \\ x - 3 & x^2 + 4 & 3x \end{vmatrix} = a_0 + a_1x + a_2x^2 + \dots - a_7x^7$$

$$x = 0 \Rightarrow \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -3 \\ -3 & 4 & 0 \end{vmatrix} = a_0$$

$$a_0 = 1(-9) + 3(4+6) = -9 + 30 = 21$$

$$39. a_1 = a_2 = a_3 = \dots a_n = 1 \Rightarrow \log a_1 = \log 1 = 0$$

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$$40. \text{Det}(B^{-1}AB) = |B^{-1}| |A| |B| = \frac{1}{|B|} |A| |B| = |A|$$

$$41. \frac{d}{dx}(\cos x^0) = -\sin x^0 \times 1^0 = \frac{-\pi}{180} \sin x^0$$

$$42. y = e^{\sin x} \Rightarrow \frac{dy}{dx} = e^{\sin x} \cos x = y \cos x$$

$$43. \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + ax - 1} - x \right) = \frac{a}{2} = \frac{2}{2} = 1$$

$$44. \lim_{n \rightarrow \infty} \left(1 - \frac{4}{x-1} \right)^{3x-1} = e^{\lim_{n \rightarrow \infty} (3x-1) \left(\wedge - \frac{4}{x-1} - 1 \right)} = e^{-12}$$

$$45. ax^2 + bx + c = a(x - \alpha)(x - \beta)$$

$$\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2} = \lim_{x \rightarrow \alpha} \frac{1 - \cos(a(x - \alpha)(x - \beta))}{(x - \alpha)^2 \times (x - \beta)^2} (x - \beta)^2 = \frac{a^2}{2} (\alpha - \beta)^2$$