

: HINTS AND SOLUTIONS :

3 (b)

$$e = n \frac{dB}{dt}$$

$$\therefore dt = \frac{nAB}{e} = \frac{50 \times 100 \times 10^{-4} \times 2 \times 10^{-2}}{5}$$

$$= 0.1 \text{ s}$$

4 (c)

$$e = \frac{d\phi}{dt} = \frac{1 - 0.1}{0.1} = \frac{0.9}{0.1} = 9 \text{ V.}$$

$$I = \frac{e}{R} = \frac{1}{100} = 0.01 \text{ A}$$

5 (c)

magnetic flux linked with coil $\phi = Bna$

$$d\phi = \phi_2 - \phi_1 = (B_2 - B_1)nA$$

$$= (0.1 - 0.05) \times 100 \times (10 \times 5 \times 10^{-4})$$

$$= 250 \times 10^{-4}$$

emf induced in coil $e = \frac{d\phi}{dt} = \frac{d\phi}{dt} = \frac{250 \times 10^{-4}}{0.05}$

$$= 0.5 \text{ V}$$

6 (d)

$\phi = nBA$, so it is independent in density.

11 (b)

$$e = -L \frac{dI}{dt} = -5 \times 2 = -10 \text{ V}$$

12 (a)

$$E = \frac{1}{2} LI^2 = \frac{1}{2} \times 50 \times 10^{-3} \times (4)^2$$

$$= \frac{1}{2} \times 50 \times 10^{-3} \times 16$$

$$= 50 \times 10^{-3} \times 8 = 0.4 \text{ J}$$

14 (a)

Inductors are connected in parallel then the equivalent

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$$

$$L_p = 1 \text{ H}$$

16 (d)

$V_p = 200 \text{ volt}, V_s = 25 \text{ volt}, I_s = 2 \text{ A}, I_p = ?$

$$\frac{V_s}{V_p} = \frac{I_p}{I_s}$$

$$I_p \frac{V_s}{V_p} \times I_s = \frac{25}{200} \times 2 = 0.25 \text{ A} = 250 \text{ mA}$$

17 (a)

$$V_s = \frac{n_s}{n_p} \times V_p = \frac{5000}{500} \times 20 = 200 \text{ V frequency}$$

remains unchanged

18 (d)

$$\frac{V_s}{V_p} = \frac{n_s}{n_p}; \frac{V_s}{V_p} = \frac{I_p}{I_s}; \frac{I_p}{I_s} = \frac{n_s}{n_p} = \frac{4}{5}$$

19 (a)

$\phi = MI, d\phi = M dI$

$$M = \frac{d\phi}{dI} = \frac{1.6}{8} = 0.2 \text{ H}$$

20 (b)

$$e = -M \frac{dI}{dt} \Rightarrow 15 \times 10^{-3} = M \times \frac{3}{10}$$

$$\Rightarrow M = 0.05 \text{ H}$$

21 (d)

$$L = \frac{\phi}{I} = \frac{50}{5} = 10 \text{ H}$$

22 (c)

$$M = \frac{e}{di/dt} = e \cdot dt/di = \frac{1000 \times 0.01}{2} = 5 \text{ H}$$

23 (b)

$e_0 = nAB\omega = naB \cdot 2\pi n$

$$= 2 \times 7 \times 10^{-5} \times 2 \times \frac{22}{7} \times 100 = 88 \text{ mV}$$

Question Numbers – 1, 7,8,9,10,15,25 – All conceptual understanding based

: HINTS AND SOLUTIONS :

- 1 (b)
The isomeric primary alcohols of $C_4H_{10}O$ are :
(1) Butan-1-ol
(2) 3-Methyl propan-1-ol
- 4 (a)
$$CH_3CHO + 2[H] \xrightarrow[\Delta]{Na(Hg)+H_2O} CH_3CH_2OH$$
- 5 (c)
$$\begin{array}{c} O \\ || \\ CH_3 - C - OH \end{array} + 4[H] \xrightarrow[\Delta]{LiAlH_4} CH_3CH_2OH + H_2O$$
- 8 (c)
Formaldehyde + Grignard Reagent \rightarrow 1° alcohol
- 10 (a)
Y on oxidation gives acetone.
Therefore Y is Propan-2-ol.
X on hydration gives Propan-2-ol. Therefore X is Propene.
- 11 (a)
During breaking of O - H bond, order of reactivity is $1^\circ > 2^\circ > 3^\circ$.
- 12 (a)
 $-CH_2OH \xrightarrow{[O]} -CHO$
- 18 (c)
1 double bond \Rightarrow Sp^2 hybridised.

All rest other other Questions are conceptually based

1 (b)

2 (c)

$$\begin{aligned} I &= \int \frac{dx}{1+e^{-x}} \\ &= \int \frac{e^x}{1+e^x} dx \\ &= \log(1+e^x) + c \end{aligned}$$

3 (d)

$$\begin{aligned} I &= \int \frac{e^x(1+x)}{\cos^2(xe^x)} dx \\ \text{Put } xe^x dx &= dt \\ e^x(1+x)dx &= dt \\ I &= \int \sec^2 t dt \\ &= \tan t + c \\ &= \tan(xe^x) + c \end{aligned}$$

4 (b)

$$\begin{aligned} I &= \int x^2 e^x dx \\ &= x^2 e^x - \int 2x e^x dx \\ &= x^2 e^x - 2x e^x + 2e^x + c \\ &= e^x(x^2 - 2x + 2) + c \end{aligned}$$

5 (b)

$$\begin{aligned} I &= \int e^x(\sin x + \cos x) dx \\ &= \int e^x \left[\sin x + \frac{d}{dx}(\sin x) \right] dx \\ &= e^x \sin x + c \end{aligned}$$

6 (b)

$$\begin{aligned} I &= \int e^x \left(\log x + \frac{1}{x} \right) dx \\ &= e^x \left(\log x + \frac{d}{dx}(\log x) \right) dx \\ &= e^x \log x + c \end{aligned}$$

7 (c)

$$\begin{aligned} I &= \int e^x \left(\frac{x+2}{(x+3)^2} \right) dx \\ &= \int e^x \left(\frac{1}{x+3} - \frac{1}{(x+3)^2} \right) dx \\ &= \int e^x \left(\frac{1}{x+3} + \frac{d}{dx} \left(\frac{1}{(x+3)^2} \right) \right) dx \\ &= \frac{e^x}{x+3} + c \end{aligned}$$

8 (b)

$$\begin{aligned} I &= \int (\sin(\log x) + \cos(\log x)) dx \\ \text{Put } \log x = t \quad dx &= e^t dt \\ I &= \int e^t (\sin t + \cos t) dt \end{aligned}$$

$$\begin{aligned}
&= \int e^t \left(\sin t + \frac{d}{dx}(\cos t) \right) dt \\
&= e^t \sin t + c \\
&= x \sin(\log x) + c
\end{aligned}$$

9

(b)

$$I = \int \left(\frac{1}{\log x} - \frac{1}{(\log x)^2} \right) dx$$

$$\text{Put } \log x = t \quad \therefore x = e^t dt$$

$$I = \int e^t \left(\frac{1}{t} - \frac{1}{t^2} \right) dt$$

$$= e^t \left(\frac{1}{t} \right) + c$$

$$= x \left(\frac{1}{\log x} \right) + c$$

10

(a)

$$I = \int e^{\sin^{-1} x} \left(\frac{x + \sqrt{1-x^2}}{\sqrt{1-x^2}} \right) dx$$

$$\text{Put } \sin^{-1} x = t$$

$$\frac{dx}{\sqrt{1-x^2}} = dt \text{ and } x = \sin t$$

$$I = \int e^t (\sin t + \sqrt{1-\sin^2 t}) dt$$

$$I = \int e^t (\sin t + \cos t) dt$$

$$I = e^t \sin t + c$$

$$I = x e^{\sin^{-1} x} + c$$

11

(c)

$$I = \int e^{\tan^{-1} x} \left(\frac{1+x+x^2}{1+x^2} \right) dx$$

$$\text{Put } \tan^{-1} = t \quad \therefore \frac{1}{1+x^2} dx = dt$$

$$= \int e^t (1 + \tan^2 t + \tan t) dt$$

$$= \int e^t (\tan t + \sec^2 t) dt$$

$$= e^t \tan t + c$$

$$= x e^{\tan^{-1} x} + c$$

12

(d)

$$I = \int \sqrt{4-9x^2} dx$$

$$= \int 3 \sqrt{\left(\frac{2}{3}\right)^2 - x^2} dx$$

$$= 3 \left\{ \frac{x}{2} \sqrt{\left(\frac{2}{3}\right)^2 - x^2} + \frac{4}{9} \sin^{-1} \left(\frac{x}{\frac{2}{3}} \right) \right\} + c$$

$$= \left\{ \frac{x}{2} \sqrt{4-9x^2} + \frac{2}{3} \sin^{-1} \left(\frac{3x}{2} \right) \right\} + c$$

13

(a)

$$I = \int \frac{dx}{x-x^2}$$

$$= \int \frac{1}{x(1-x)} dx$$

$$\text{Let } 1 = A(x) + B(1-x)$$

$$\begin{aligned}
A &= 1, & B &= 1 \\
I &= \int \frac{x + (1-x)}{x(1-x)} dx \\
&= \int \frac{1}{1-x} dx + \int \frac{1}{x} dx \\
&= -\log(1-x) + \log x + c \\
&= \log x - \log(1-x) + c
\end{aligned}$$

14

(a)

$$\begin{aligned}
I &= \int \frac{dx}{x^2 - 5x + 6} \\
&= \int \frac{1}{(x-2)(x-3)} dx \\
\text{Let } 1 &= A(x-2) + B(x-3) \\
A &= 1, & B &= -1 \\
I &= \int \frac{(x-2) - (x-3)}{(x-2)(x-3)} dx \\
&= \int \frac{1}{x-3} dx - \int \frac{1}{x-2} dx \\
&= \log(x-3) - \log(x-2) + c \\
&= \log\left(\frac{x-3}{x-2}\right) + c
\end{aligned}$$

15

(d)

$$\begin{aligned}
I &= \int \frac{\sin x}{\cos x(4 + \cos x)} dx \\
\text{Put } \cos x &= t \Rightarrow \sin x dx = -dt \\
I &= \int \frac{-dt}{t(4+t)} \\
\text{By Partial Fraction} \\
I &= \int \left(\frac{-1}{4t} + \frac{1}{4(4+t)} \right) dt \\
I &= \frac{1}{4} (\log |4+t| - \log |t|) + c \\
I &= \frac{1}{4} \log \left| \frac{4 + \cos x}{\cos x} \right| + c
\end{aligned}$$

16

(c)

$$\begin{aligned}
I &= \int \frac{\log x}{x(2 + \log x)(3 + \log x)} dx \\
\text{Put } \log x &= t \Rightarrow \frac{dx}{x} = dt \\
I &= \int \frac{t}{(t+2)(t+3)} dt \\
\text{By Partial Fraction} \\
t &= A(t+3) + B(t+2) \\
\text{Put } t &= -3 \Rightarrow B = 3 \\
I &= \int \left(\frac{-2}{t+2} + \frac{3}{t+3} \right) dt \\
I &= -2 \log |t+2| + 3 \log |t+3| + c \\
I &= 3 \log |\log x + 3| - 2 \log |\log x + 2| + c
\end{aligned}$$

17

(a)

$$\begin{aligned}
I &= \int \frac{5x + 2}{x^2 - 3x + 2} dx \\
I &= \int \frac{5x + 2}{(x-1)(x-2)} dx \\
\text{By Partial Fraction}
\end{aligned}$$

$$5x + 2 = A(x - 2) + B(x - 1)$$

$$\text{Put } x = 1 \Rightarrow A = -7$$

$$\text{Put } x = 2 \Rightarrow B = 12$$

$$\text{Put } x = 2 \Rightarrow B = 12$$

$$I = \int \left(\frac{12}{x-2} - \frac{7}{x-1} \right) dx$$

$$I = 12 \log |x-2| - 7 \log |x-1| + c$$

18

(c)

$$\begin{aligned} I &= \int e^x \left(\frac{x}{(x+1)^2} \right) dx \\ &= \int e^x \left(\frac{1}{x+1} - \frac{1}{(x+1)^2} \right) dx \\ &= \int e^x \left[\frac{1}{x+1} + \frac{d}{dx} \left(\frac{1}{(x+1)^2} \right) \right] dx \\ &= \frac{e^x}{x+1} + c \end{aligned}$$

19

(a)

$$\begin{aligned} I &= \int e^x (x^2 + 2x + 5) dx \\ &= \int e^x \left[(x^2 + 5) + \frac{d}{dx} (x^2 + 5) \right] \\ &= e^x (x^2 + 5) + c \end{aligned}$$

20

(c)

$$\begin{aligned} I &= \int e^x \sin x dx \\ &= e^x \sin x - \int e^x \cos x dx \\ I &= e^x \sin x - e^x \cos x - \int e^x \sin x dx \\ 2I &= e^x (\sin x - \cos x) + c \\ I &= \frac{e^x}{2} (\sin x - \cos x) + c \end{aligned}$$

21

(a)

$$\begin{aligned} I &= \int \frac{1}{2 + 3 \sin^2 x} dx \\ &= \int \frac{\sec^2 x}{2 \sec^2 x + 3 \tan^2 x} dx \\ &= \int \frac{\sec^2 x}{5 \tan^2 x + 2} \\ \text{Put } \tan x &= t, \quad \sec^2 x dx = dt \\ I &= \int \frac{1}{5t^2 + 2} dt \\ &= \frac{1}{5} \int \frac{1}{t^2 + \frac{2}{5}} dt \\ &= \frac{1}{5} \cdot \frac{\sqrt{5}}{\sqrt{2}} \tan^{-1} \left(\frac{\sqrt{5}t}{\sqrt{2}} \right) + c \\ &= \frac{1}{\sqrt{10}} \tan^{-1} \left(\frac{\sqrt{5} \tan x}{\sqrt{2}} \right) + c \end{aligned}$$

22

(a)

$$\int \frac{sdx}{5 + 4 \cos x}$$

Put $\tan x/2 = t,$

$$\begin{aligned}
 dx &= \frac{2dt}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2} \\
 &= \int \frac{\frac{2dt}{1+t^2}}{\frac{5+5+t^2 4(1-t^2)}{1+t^2}} = 2 \int \frac{1}{9+t^2} dt \\
 &= \frac{2}{3} \times \tan^{-1}\left(\frac{t}{3}\right) + c = \frac{2}{3} \tan^{-1}\left(\frac{1}{3} \tan \frac{x}{2}\right) + c
 \end{aligned}$$

23

(b)

$$\log x = t$$

$$\frac{1}{x} dx = dt$$

$$I = \int \log t dt$$

$$= t. [\log t - 1] + c$$

(Integration by part)

$$= \log x [\log(\log x) - 1] + c$$

24

(c)

$$I = \int \frac{1}{x^2 + 4x - 5} dx$$

$$= \int \frac{1}{(x+2)^2 - 3^2} dx$$

$$\frac{1}{6} \log \left| \frac{x-1}{x+5} \right| + c$$

25

(d)

$$I = \int x^n \log x dx$$

$$I = (\log x) \frac{x^{n+1}}{n+1} - \int \frac{1}{x} \frac{x^{n+1}}{n+1} dx$$

$$I = (\log x) \frac{x^{n+1}}{n+1} - \frac{1}{n+1} \int x^n dx$$

$$I = \frac{(\log x)x^{n+1}}{n+1} - \frac{x^{n+1}}{(n+1)^2} + c$$

Mathematics

: HINTS AND SOLUTIONS :

